تصميم عدسة كهروستاتيكية ثلاثية الأقطاب بطريقة كثافة الشحنة

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الخلاصـــــة

تركز هذا البحث على تصميم عدسة كهروستاتيكية ثلاثية الاقطاب اسطوانية الشكل متحدة المركز ومفصولة بفتحة هوائية . تم حل معادلة لابلاس بطريقة كثافة الشحنة في مجال بصريات الجسيمات المشحونة اللانسبية وبغياب تأثيرات شحنة الفضاء إن توزيع الجهد المحوري لعدسة ثلاثية كهروستاتيكية تم حسابه باستخدام كثافة الشحنة الموزعة على الاقطاب، الناتجة من تسليط فولتية معينة على الاقطاب اعتمادا على قانون كولوم. وتم حل معادلة الأشعة المحورية للجهد المحسوب لإيجاد مسار الجسيمات المشحونة المارة من خلال العدسة بالاستعانة بطريقة رنج-كوتا. وحسبت الخواص البصرية للعدسة تحت شروط التكبير الصفري والتكبير اللانهائي.

Computer-Aided Design of an Einzel Lens by the Charge Density Method

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Abstract

This work has been concentrated on designing three electrodes electrostatic einzel lens whose electrodes are equidiameter cylindrical in shape separated by an air gap. The charge density method for solving Laplace's equation has been carried out in the field of non-relativistic charged-particle optics under the absence of space-charge effects. Potentials have been determined anywhere in space by using Coulomb's law. The paraxial ray equation has been solved with the aid of the computed axial potential distribution in order to determine the trajectory of the charged-particles beam along the lens field using Runge- Kutta method. The optical properties of the einzel lens have been investigated under zero and infinite magnification conditions.

Introduction

Many computer programs have been developed for solving problems in electrostatic charged particle optics. Nearly all of the programs are based on one of the following methods (a) Finite Difference Method (FDM) (b) Finite Element Method (FEM) and (c) Boundary Element Method (BEM). The BEM is also known as charge density method (CDM) or boundary charge method (BCM); see for example [1].

A new method is presented of solving Laplace's equation for equidiameter coaxial cylinders separated by a finite distance. This method has been found to give accurate results, efficient in the use of computer time and storage, and applicable to a wide range of lens configurations. The charge density method is a particular example of Boundary Element Method (BEM). In most of the published work the lenses that are used for this purpose have been divided into N-rings; these rings are of variable width and are made narrower near the gap, where the charge density changes most rapidly [2,3]. However, in the present work the system of cylinders under applied potential has been replaced by a system of charged rings, which have the same width as illustrated in figure 1.

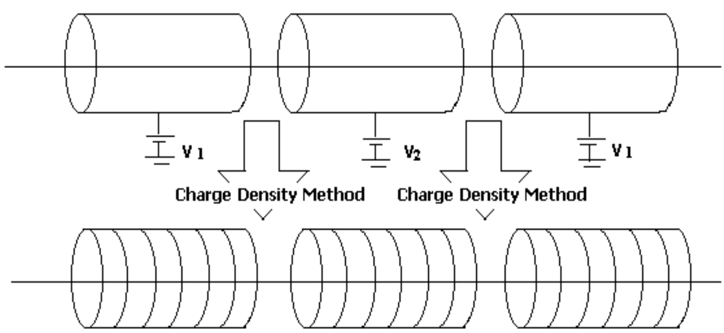

Figure 1. Replacing a series of cylinders under applied potentials with a series of charged rings [3].

Method

To solve the problem, the cylinders have been divided into N rings; each ring carries a charge Q_i (i =1, 2, 3,…, N) which contributes to the potentials of all the rings. The potential of the ith ring can be expressed as a combination of the contributions from all charged rings [3]. Consider the lens cylinders of radius r_c and length 10 r_c [4]. The combined charge densities on the surfaces of the cylinders are $\sigma_i = Q_i / 4\pi r_c \Delta z_i$, where Δz_i represents the width of the ith rings. If there are no other charges present then the potential at any point z in space is given by,

$$U(r_c, z) = \frac{1}{\pi \varepsilon_o} \sum_{\substack{i=1 \\ j \neq i}}^{N} \sigma_i k_i K(k_i^{\,2}) \Delta z_i \qquad \textbf{......(1)}$$

where
$$k_i = \frac{2r_c}{\left[4r_c^{\,2} + \left(z_i - z\right)^2\right]^{1/2}}$$

and $K(k^2{}_i)$ is the complete elliptic integral of the first kind which can be evaluated by the use of the following polynomial approximation [5],

$$K(k_i) = a_0 + a_1 H + a_2 H^2 + a_3 H^3 + a_4 H^4 + (b_0 + b_1 H + b_2 H^2 + b_3 H^3 + b_4 H^4)\ln(1/H)$$
….. (2)

where H =1-k_i^2 which is a dimensionless factor. The potential V_j at a point C on the ith element is due to a constant charge density σ on each element, which is uniformly distributed around a circle of radius r_c . The potential V_j is given by the following expression [6],

$$V_j = \sum_{i=1}^{N} A_{ji} \sigma_i \qquad \textbf{...... (3)}$$

where A_{ji} is a square matrix element. The above set of equations may be reduced to the following simple matrix equation,

$$V = A \cdot \sigma \qquad \text{......} \; \mathbf{(4)}$$

The charge density σ is mathematically considered a column vector. In applying this procedure to the cylinder problem one may take same values of the voltage applied on the first and third electrodes and different value for second electrode, the column vector σ is then obtained by inverting the matirx A [2,7]. Hence, from equation (4), $\sigma = A^{-1} \cdot V$ ....**(5)**

In the present work an iterative procedure is used to get the inverse of matrix A with the aid of a computer program based on LU-Factorization method [8]. To evaluate the elements of A one needs to know the potential at the strip j caused by a uniform charge density σ_i in the strip i. The matrix element A_{ji} is given by [4],

$$A_{ji} = \frac{k_{ji} \Delta z_i}{\pi \varepsilon_0} K(k_{ji}{}^2)$$

....**(6)**

where $k_{ji} = \dfrac{2r_c}{\left[4r_c{}^2 + z_{ji}{}^2\right]^{1/2}}$

and $z_{ji} = \left| \overline{z_i} - \overline{z_j} \right|$

$\overline{z_i}$ and $\overline{z_j}$ being the mid point of the ith and jth ring respectively; they are given by $\overline{z_i} = (z_{i+1} + z_{i-1})/2$ and $\overline{z_j} = (z_{j+1} + z_{j-1})/2$. It should be noted that when j is equal to i the elliptic integral (equation 2) will be infinite and a singularity in the potential V is caused but not in A_{ii} itself.

The Trajectory equation of motion of a charged particle travelling at a non-relativistic velocity in an electric field near the axis of a cylindrically symmetric system can be reduced to the following equation [9,10],

$$\frac{d^2R}{dz^2}+\frac{U'}{2U}\frac{dR}{dz}+\frac{U''}{4U}R=0$$

(7)

where U' and U'' are the first and second derivatives of the axial potential U respectively. R represents the radial displacement of the beam from the optical axis z. The most important aberrations in an electron-optical system are spherical and chromatic aberration. Thus the present work has been focused on determining these two aberrations for an einzel electrostatic lens. The spherical and chromatic aberration coefficients are denoted by Cs and Cc respectively. In the present investigation the values of Cs and Cc are normalized in terms of the image side focal length. The spherical aberration coefficient Cs and the chromatic aberration coefficient Cc referred to the image/object side are calculated from the following equations [5].

$$Cs=\frac{U^{-1/2}}{16R'^4}\int_{Zo}^{Zi}\left[\frac{5}{4}\left(\frac{U''}{U}\right)^2+\frac{5}{24}\left(\frac{U'}{U}\right)^4+\frac{14}{3}\left(\frac{U'}{U}\right)^3\frac{R'}{R}-\frac{3}{2}\left(\frac{U'}{U}\right)^2\frac{R'^2}{R}\right]\sqrt{U}R^4dz$$

(8)

$$Cc=\frac{U^{1/2}}{R'^2}\int_{Zo}^{Zi}\left(\frac{U'}{2U}R'R+\frac{U''}{4U}R'\right)U^{-1/2}dz$$

(9)

where $U = U(z)$ is the axial potential, the primes denote derivative with respect to z, and $U_i = U(z_i)$ is the potential at the image where $z = z_i$. The integration given in the above equations are executed by means of Simpson's rule [5,11]. In the present work, equations (8) and (9) have been used for computing Cs and Cc in the image side under various magnification conditions.

Results and discussion

Due to the limited number of elements of the matrix A each electrode has been divided into equal rings the charge density distribution due to the applied voltages (figure 2) on the electrodes region each point on the graph represents a uniform charge density for a particular ring. Within the air gap there is no charge density due to the rings situated at the two terminals of each electrode, which are at a close proximity to air. Further more, the charge density on the 1st and 3rd electrodes are similar in shape at the lower voltage and its values is less than the charge density on the 2nd electrode at the higher voltage.

It is assumed that a potential V1=10 volts is applied on the 1st and 3rd electrodes of figure 1 and V2=18volts on the 2nd electrode. The diameter D and length L of each cylinder are 10 r_c. The axial potential distributions are shown in figure 3. There is a field-free region when E (z)=0 outside the lens boundaries. It is seen that the gradient of the curve at the region of the two gaps close to the higher voltage electrode increases while that close to the lower voltage electrode decreases. The potential U (z) at this maximum point of the curve (Z =0.0) equals to 18V, which is the potentials applied on the 2nd electrode. Furthermore, this maximum point can be used as a criterion for the classification of the lens whether it is symmetrical or asymmetrical. Within the air gap region, the potential on the side of the lower voltage electrode penetrates the hollow cylindrical electrode and its gradient diminishes at a point (Z=-2.5mm). The value of the potential at this point is equal to the voltage applied on the corresponding electrode (i.e. U(z)=10V). On the other hand, the potential on the side of the higher voltage electrode penetrates the hollow electrode region and its gradient diminishes at a point (Z=+2.5mm) where its value equals to that of the applied voltage.

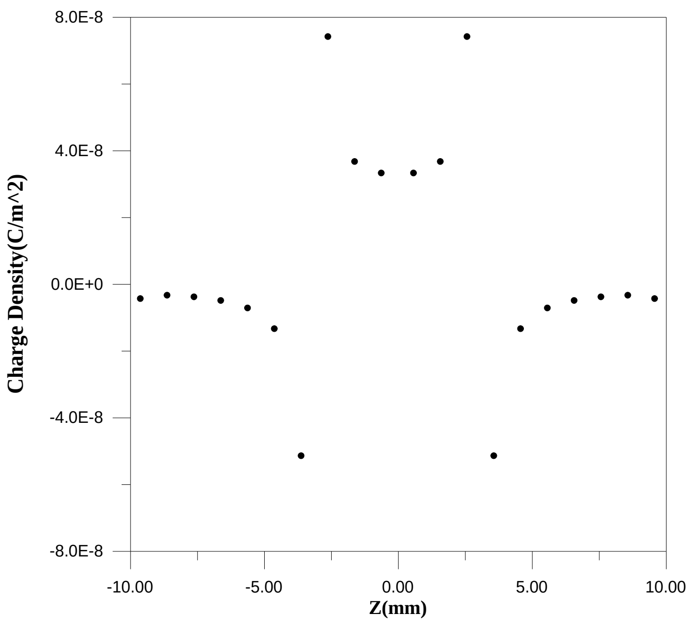

Figure2. The charge density distribution on three electrodes .

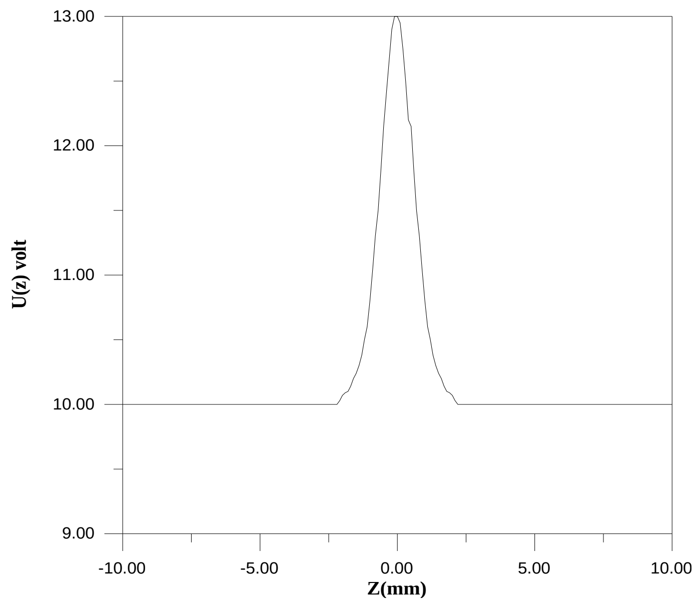

Figure 3. The axial potential distribution on the einzel lens .

The electron beam path along the electrostatic lens field under zero magnification condition and accelerating mode of operation has been considered. Figure 4 shows the trajectories of an electron beam traversing the electrostatic lens field. Computations have shown that as the beam emerges from the lens field it converges towards the optical axis provided that V1/V2 does not exceed 0.55. In this case the beam intersects the optical axis once. However, as V2/V3 exceeds 1.8, the beam emerges divergent; this is due to the increase of the lens refractive power with the increase of the voltage ratio. The distortion of the trajectory increases in the regions between the 1st and 2nd electrodes and in the region between the 2nd and 3rd electrodes at the entrance side of the beam.

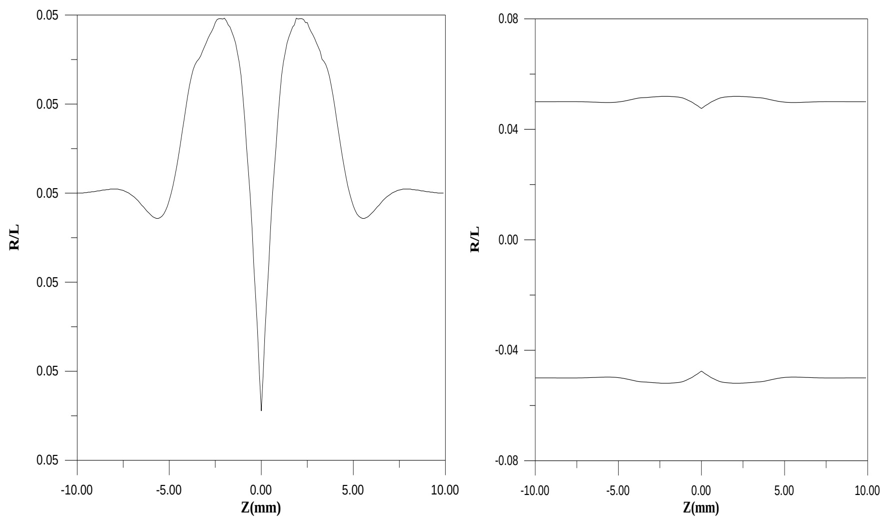

Figure 4. The electron beam trajectory in the electrostatic einzel lens under zero magnification condition

The spherical and chromatic aberration coefficients have been given Considerable attention in the present work since they are the two most important aberrations in electron optical system.

Under zero magnification it is seen that as the voltage ratios V2/V1 increases, the relative spherical aberration coefficient Cs/f_i decreases but the relative chromatic aberration coefficient Cc/f_i respectively increasing linearly. At (V2/V1 =8) the value of Cs/f_i has a minimum value equal (66.5079) .When (V2/V1 =2) the value of Cc/f_i has a minimum value equal (3.048119), as shown in figure (5).

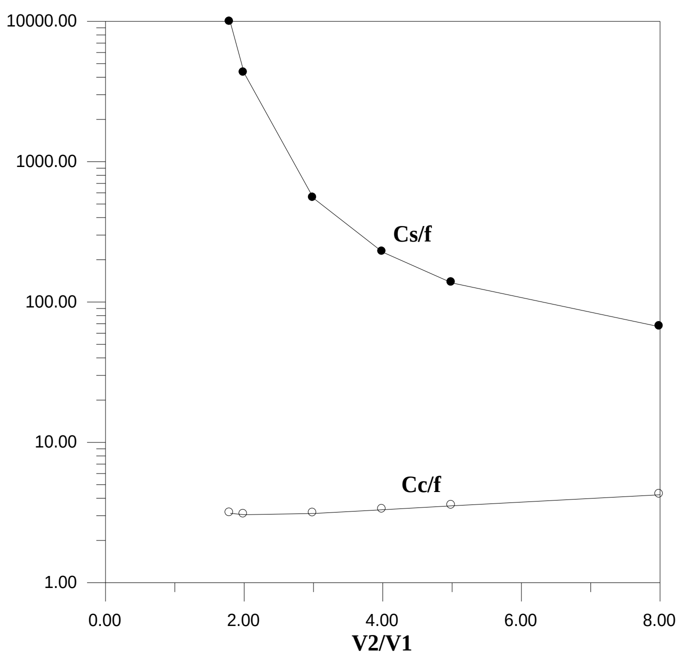

Figure 5.The spherical and chromatic aberration coefficients under zero magnification condition

Under infinite magnification, it seen that as the voltage ratios (V2/V1) increasing , the relative spherical aberration coefficients Cs/f_o are decreases .At the voltage ratio (V2/V1 = 1.8) the spherical aberration coefficients has a minimum value (Cs/f_o =32.1375) , also the relative chromatic aberration coefficient Cc/f_i are decreases and at the voltage ratio (V2/V1 =8) the chromatic aberration coefficients has a minimum value (Cc/f_o = 1.21875) ,as shown in figure (6).

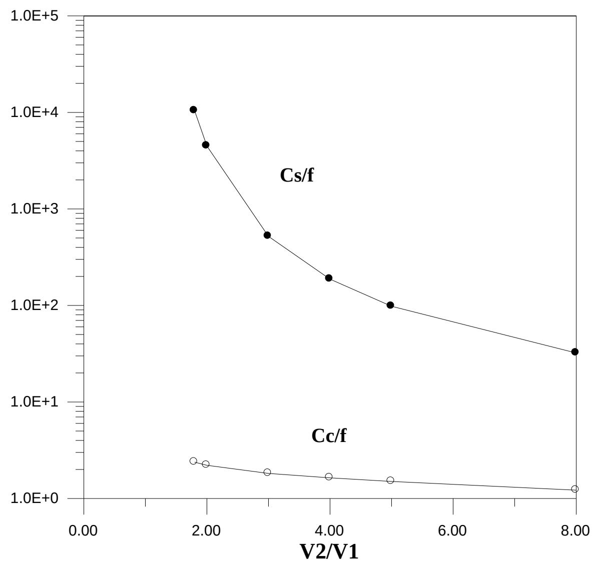

Figure 6.The spherical and chromatic aberration coefficients under infinite magnification condition

Conclusions

The charge density method uses on the design of electrostatic lenses appears to be an excellent tool in the field of electron-optical design. The cylindrical einzel lens that has been designed by the above method is found to have different optical properties depending upon various geometrical parameters in addition to the mode of operation. For instance under zero magnification mode of operation this lens did not exhibit acceptable properties from the electron-optical point of view. However, in the infinite magnification mode of operation the lens performance was found to be excellent. The optical properties are highly dependent on the geometrical factors of the lens such as the radii and the lengths of the cylinders. Thus, one could now apply the charge density method on designing various types of electrostatic lenses.

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